

Chain Rule

The derivative of $f(g(x))$ is $f'(g(x)) \cdot g'(x)$.

$$1. \ y = \sqrt{x^3 + 7}$$

$$y = (x^3 + 7)^{1/2}$$

$$y' = \frac{1}{2}(x^3 + 7)^{-1/2} (3x^2)$$

$$y' = \frac{3x^2}{2(x^3 + 7)^{1/2}}$$

$$2. \ y = \frac{2}{(5x+1)^2}$$

$$y = 2(5x+1)^{-2}$$

$$y' = -4(5x+1)^{-3} (5)$$

$$y' = \frac{-20}{(5x+1)^3}$$

$$3. \ y = \tan 6x$$

$$y' = \sec^2(6x) \cdot 6$$

$$y' = 6 \sec^2(6x)$$

$$4. \ y = (x^2 + 5)^3 (x^3 - 1)^4$$

$$y' = 3(x^2 + 5)^2 (2x) (x^3 - 1)^4 + (x^2 + 5)^3 4(x^3 - 1)^3 (3x^2)$$

$$y' = 6x(x^2 + 5)^2 (x^3 - 1)^4 + 12x^2 (x^2 + 5)^3 (x^3 - 1)^3$$

$$y' = 6x(x^2 + 5)^2 (x^3 - 1)^3 [(x^3 - 1) + 2x(x^2 + 5)]$$

$$y' = 6x(x^2 + 5)^2 (x^3 - 1)^3 [x^3 - 1 + 2x^3 + 10x]$$

$$y' = 6x(x^2 + 5)^2 (x^3 - 1)^3 (3x^3 + 10x - 1)$$

$$5. \ y = \cos^3 x^2$$

$$y = (\cos x^2)^3$$

$$y' = 3(\cos x^2)^2 (-\sin x^2) (2x)$$

$$y' = -6x \cos^2(x^2) \sin(x^2)$$

$$6. \ y = \sqrt{\sin(6x)}$$

$$y = (\sin(6x))^{1/2}$$

$$y' = \frac{1}{2}(\sin(6x))^{-1/2} (\cos(6x)) (6)$$

$$y' = \frac{3 \cos(6x)}{(\sin(6x))^{1/2}}$$

$$7. \ y = e^{\tan x}$$

$$y' = e^{\tan x} \cdot \sec^2 x$$

$$8. \ y = 3^{x^2}$$

$$y' = 3^{x^2} \cdot \ln 3 \cdot 2x$$

$$\boxed{\frac{d}{dx} e^u = e^u \cdot u'}$$

$$\boxed{\frac{d}{dx} a^u = a^u \cdot \ln a \cdot u'}$$